NUCLEAR PHYSICS 24/02/2026

Last week:

H atom spectrum. Lyman, Balmer etc. series.
Experiment is measureng a quantity of a discharges gas.

$$\tilde{\nu} \sim R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

m, n integers.
$\tilde{\nu}(+) \Rightarrow n > m$.

Via this you can observe any frequency in H.
Every line you can observe is given by this formula.

Reidhburg combinatory principle:

m = constant
change n and substract from each other (the $\tilde{\nu}$)'s are also a line in the spectrum.

JJ Thompson try to manipulate the cathode rays with $\vec{E}$ field and $\vec{B}$ field. What is the result.

Slide: "Elektro atiradimas"

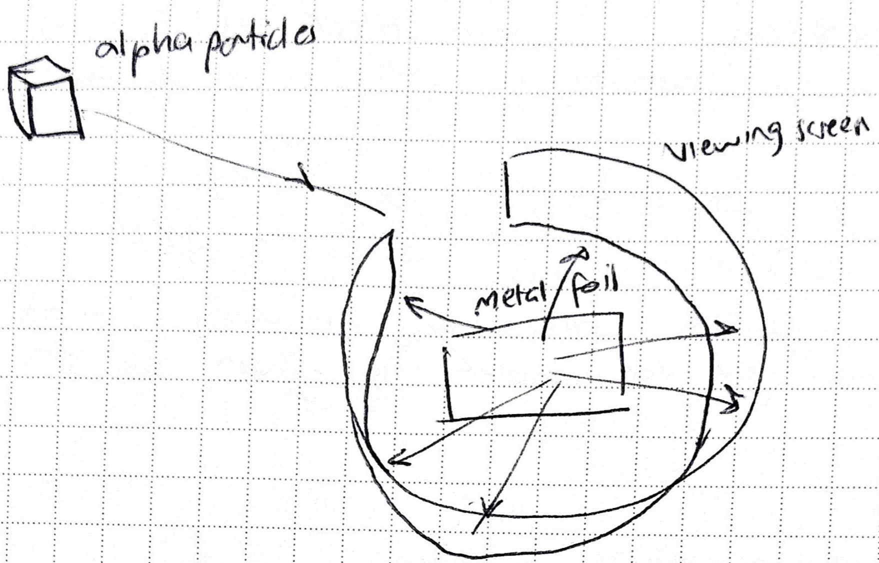
In the experiment particles are "saçılmak"
Go through the theory JJ went through (like mass/charge etc.) while discovering electron.

Also check the Oil Drop exp.

RUTHERFORD EXPERIMENT: Planetary motion of particles

1903 H. Nagaoke – planetory model

1906 E. Rutherford – starts testing and confirms in 1911
First particle physics experiment actually.



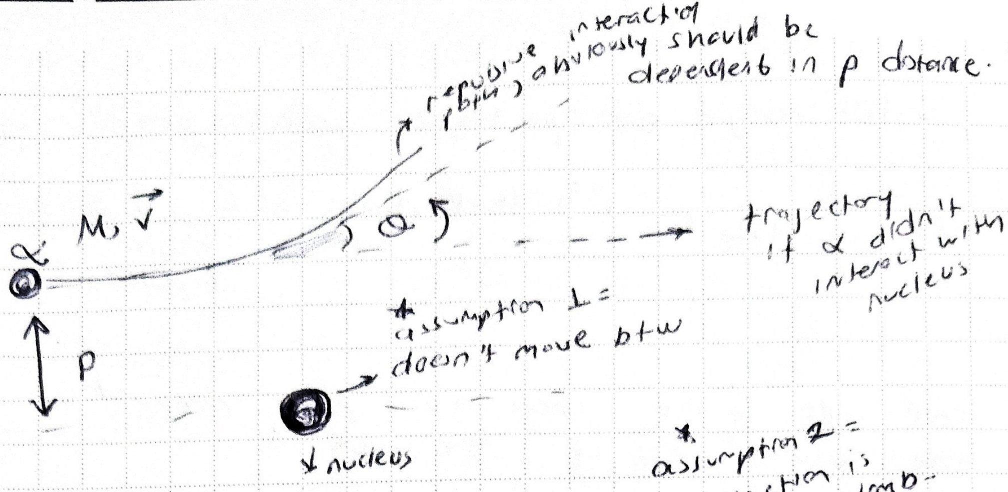
Type of materials when an energetic
particle hits them, they splash photons.
What are they called
Subs.. s... something.

α particle ^4_2He

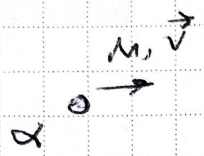
α particle has identical energy from
radioactive nuclei!!!
Well-defined energy.

^4_2He is heavy, but Gold nuclei is also
big.

The experiment compares two
scenarios:
a uniform sheet of positive charges vs.
some nuclei with planetary-like motion
forming the sheet.



p = impact parameter



$$F = \frac{2ze^2}{4\pi\varepsilon_0 r^2}$$

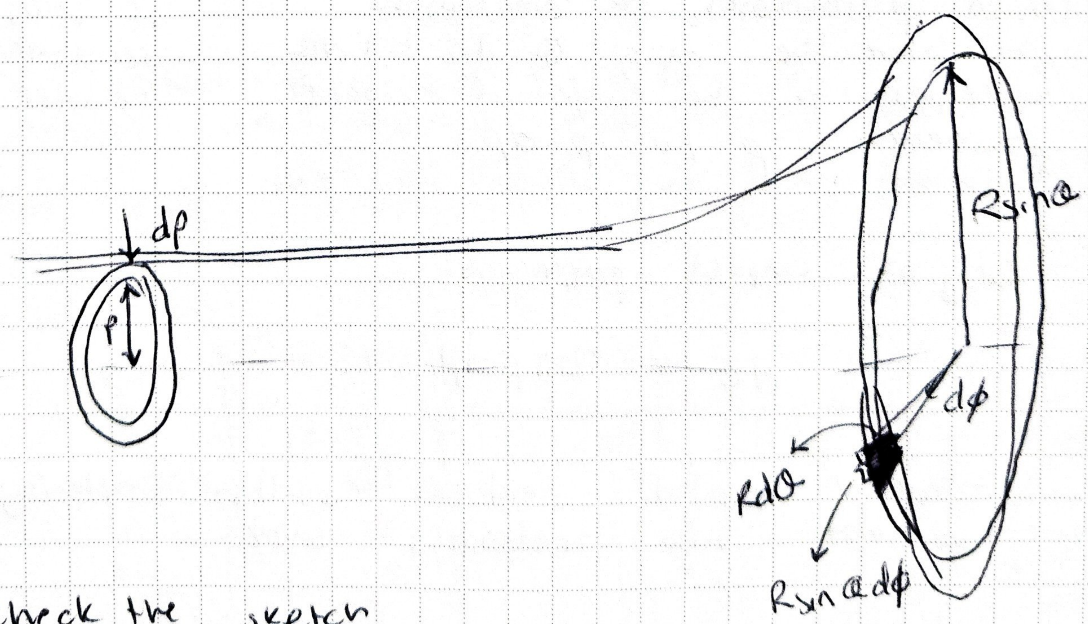
Now consider "Kepler problem" in order to derive formulation for trajectory of α.

DERIVE THIS!!!!

$$\mathrm{ctg}\,\frac{\Theta}{2} = 2\pi\varepsilon_0 \frac{Mv^2 p}{Ze^2}$$

is this cotangent?

We would like a distribution of angles with parameters of the alpha particle.



Check the sketch from slides!

Solid angle of particles with $p \in p$, $p + dp$

$$d\Omega = \frac{R^2 \sin\theta \, d\theta \, d\phi}{R^2}$$

solid angle ELEMENT

solid angle is the area on a sphere compared to the whole area of the sphere.

$$d\Omega = 2\pi \sin\theta \, d\theta$$

(is this still the element? Ω or $d\Omega$)

Probability of scattering at θ, $\theta + d\theta$

$$\frac{dN}{N} = l \cdot n \cdot d\sigma_\theta$$

particles for a particular angle

tot num of particles

differential cross section

why a cross section? because of units! Check it out!

$$\frac{dI}{I} = \varepsilon c l$$

nucleus is very likely to scatter at this angle is High!
if σ_θ is high?
If low, scattering likeliness is also low!

! when p increases, θ decreases. you have the ring θ to $\theta + d\theta$ corresponds to the range of impact parameters. p to $p - dp$.

Ring of impact parameters:

$$d\sigma = 2\pi p \, dp$$

thinner metal reduces multiple scattering so that this statistics work.

If we only count single count collusions!

$$dN_\theta = \ell \cdot n \cdot N \cdot d\sigma_\theta$$

no double scattering events.

Trajectory -

$$p^2 = \left(\frac{ze^2}{2\pi\varepsilon_0 M v^2}\right) \cdot ctg^2\frac{\theta}{2}$$

differentiate.

kotanjent. $\left(\frac{\cos}{\sin}?\right)$ idk.

$$p\,dp = \left(\frac{ze^2}{2\pi\varepsilon_0 M v^2}\right)^2 \frac{ctg\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} d\left(\frac{\theta}{2}\right)$$

$$d\theta = \frac{d\Omega}{2\pi\sin\theta} = \frac{d\Omega}{4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

plug this into the probability of scattering $(\ell \cdot n \cdot d\sigma_\theta)$

$$dN_\theta = \ell \cdot n \cdot N \left(\frac{ze^2}{4\pi\varepsilon_0 \underbrace{Mu^2}_{\text{notice, double KE}}}\right)^2 \frac{d\Omega}{\sin^4\frac{\theta}{2}} \quad *$$

* Rutherford Formula for scattering probability!!!!

You obtain a constat from this expression via something. Idk what or how tho.

Also compare the $\theta - dN_\theta$ relations. Analyse.

RESULT

θ, deg	dN_θ	$dN_\theta \cdot \sin^4\frac{\theta}{2}$
15	132000	38.4
30	7800	35
45	1435	30.8
60	477	29.8
105	69	27.5
150	33	28.5

check the $dN \sin^4(\theta)$ vs. θ graph from slides as well.

very important!!

$$\frac{dN}{N} = f n \sigma$$

cross section of nucleus.

You can estimate nucleus radius as well...

This was Rutherford type of model.

One problem with planetary-like model of atoms is leading to the use of Maxwell. Maxwell tells, spinning of electrons around nuclei should be forming and E field. Therefore (due to centrepedal force so an acceleration) an B field. So atom should be emitting. But it doesn't...

There's also uncertainty as well.

Around 1913's, it is started to be discussed that atoms have levels, descreate and well-defined.

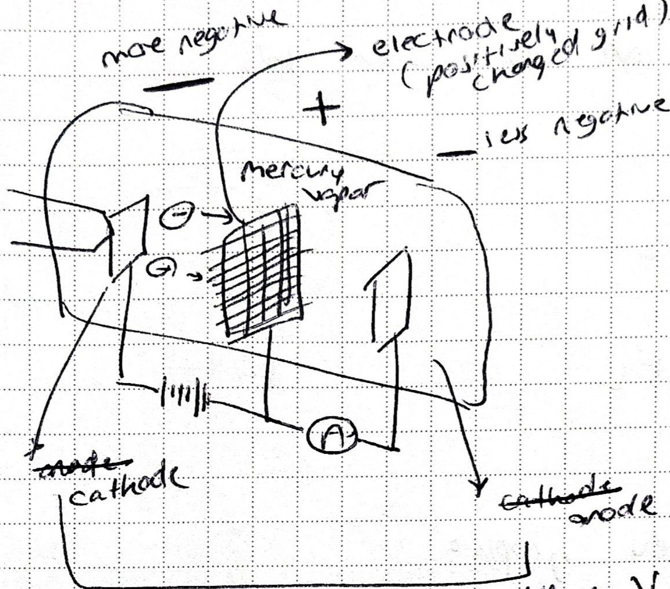
Frank - Hertz Experiment.

Assumption:

e^- in atoms have discreet energies.

$E_1, E_2, E_3 \ldots$ etc.

Frank-Hertz tests this.



there's a potential diffence V.

1st excitation = $E_2 - E_1$

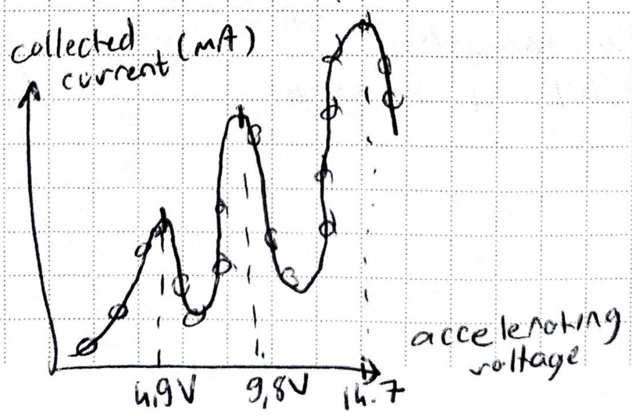
as we have enough energy satisfied to bring from lower to higher level

Increase voltage↑
Current↑

HOWEVER,

! when you reached the energy, between the electrode and the less negative anode, electron goes to the electrode.

★ So number of electrons reaching DROP as excitation occurs!!!!



Frank-Hertz Data for Mercury.

4.9V is the dropping point.

At dropping points, there's a collusions of electrons between anode and grid of some energys. Electrons end up going to the positive grid.

We indeed proven, electrons have discreate energy.

* 4.9eV is the first excitation energy for Mercury.

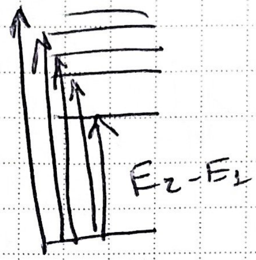
What further we can do with this experment?

We have

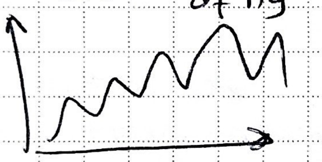
$E_2 - E_1$

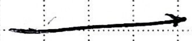
but also,

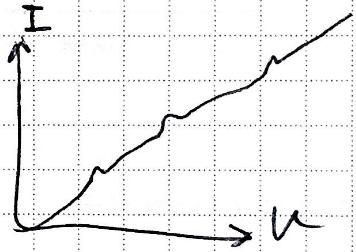
$E_3 - E_1$



Now with the prev graph =







signals lose.
BUT, higher probability to obtain all excitations.

Reduce pressure = Measure

$E_2 - E_1$
$E_3 - E_1$

Now we can also measure IONIZATION ENERGY of the atom

$E_{ionization} = 13,6 eV$ (H-atom)

N Bohr's model of atom

Postulates:

i) e^-'s are in fixed orbits with energies $E_1, E_2, E_3 \ldots$ No absorption or emission takes place when orbitting.

ii) in the H atom, e^- angular momentum is

$$L = n\hbar \quad , \quad n = 1, 2, 3, \ldots$$

Planck: $E_n = n\hbar\omega$ $\quad [\hbar] = J \cdot s$ (dim of angular momentum)

Bohr: $L = n\hbar$

when changing orbits, atom will emit or absorb ~~energy~~ light, related to energy differences of the orbitals.

iii) Atom radiates/absorbs light when changing orbits and the frequency is:

$$\hbar\omega_{mn} = E_n - E_m$$

In a circular orbit:

$$F = \frac{ze^2}{4\pi\varepsilon_0 r^2} \qquad a = \frac{v^2}{r}$$

II Newton Law

$$F = ma \qquad \frac{mv^2}{r} = \frac{ze^2}{4\pi\varepsilon_0 r^2}$$

Full energy of electron:

$$E = T + U \qquad T = \frac{1}{2}mv^2$$

$$U = -\frac{ze^2}{4\pi\varepsilon_0 r}$$

o - - o bound (−) total energy

$$F_{tot} = \frac{mv^2}{2} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

substitute = $\frac{Ze^2}{4\pi\varepsilon_0 r^2}$

$$E_{tot} = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

Now we will investigate how far e^- can be from the nucleus.

2nd Postulate = $p = mv \Rightarrow I\omega$ in circular motion =

$$L = I\omega = mr^2\omega$$

$$= n\hbar; \quad \omega = \frac{v}{r}$$

$$L = mr^2\omega = n\hbar = mvr$$

$$\frac{mv^2}{r} = \frac{rmv^2}{r^2 \omega^2} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$\frac{1}{r^2}\frac{1}{M}\underbrace{m^2v^2r^2}_{L^2} = \frac{Ze^2}{4\pi\varepsilon_0 r} = \frac{1}{r^2}\frac{1}{M}n^2\hbar^2$$

radius of the orbit ranked n =

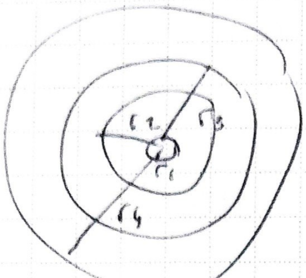
$$r_n = \frac{4\pi\varepsilon_0 n^2\hbar^2}{mZe^2} = \underbrace{4\pi\varepsilon_0\frac{\hbar^2}{mZe^2}}_{\text{unit is a length.}} n^2$$

It is the bohr radius!!!

BOHR RADIUS $r_B = 4\pi\varepsilon_0 \frac{\hbar^2}{me^2}$

0,53 Å

$$r_n = r_B \cdot n^2 \qquad (Z=1)$$



$r_1 = r_1$
$r_2 = 4r_1$
$r_3 = 9r_1$
$\vdots$
$r_6 = 36r_1$

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r_n}$$

$$E_n = -\frac{mZ^2e^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \cdot \frac{1}{n^2}$$

3rd Postulate =

$$\hbar\nu_{nk} = \hbar c \bar{\nu}_{nk} = 2\pi\hbar c \tilde{\nu}_{nk} = E_n - E_k$$

$\downarrow$
freq from orbit n to k

$$\tilde{\nu}_{nk} = \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{mZ^2e^4}{4\pi\hbar^3 c}\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$

$$\nu = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$ Ballmer Formula!

Bohr's end up clone with calculating Rayleigh constant!!!
or
Reidburg idk

From Bohr:

$$R = \frac{1}{(4\pi\varepsilon_0)^2} \frac{me^4}{4\pi\hbar^3 c} \dots$$

$$R^{theory} = 109737{,}303 \text{ cm}^{-1}$$

$$R^{exp} = 109677{,}581 \text{ cm}^{-1}$$

accuracy = $6 \cdot 10^{-4}$

Can I do better? said Bohr.

The error was due to mass, gravity etc.

Consider motion of nucleus:
reduced mass.

$$m' = \frac{m}{1 - \frac{m}{M}}$$

m = e^- mass
M – nucleus mass

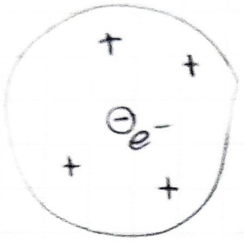
by replacing $m_e \rightarrow m'$
accuracy is $\sim 10^{-7}$

For other nuclei: for other atoms ($Z \neq 1$, $A \neq H$)

$$E_n = Z^2 \cdot (13.6 eV) \frac{1}{n^2}$$

EXERCISES

I Find the radius of H atom given the ionization energy of 13.6eV. Apply Thompson's pudding model.



SOLUTION

electron – in the center of a charged ball

Inside el. field $E_1 = \frac{er}{4\pi\varepsilon_0 R^3}$ => Gauss Theorem.

flux through the surface hypothetical surface.

$$\Phi = EA$$
$$= \frac{1}{\varepsilon_0} q'$$

Outside el. field

$$E_2 = \frac{e}{4\pi\varepsilon_0 r^2}$$

Now we know the force applies on electron.

$$W_{ionization} = \frac{e^2}{4\pi\varepsilon_0}\left[\int_0^R \frac{r}{R^3}dr + \int_R^\infty \frac{dr}{r^2}\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{R^3}\frac{r^2}{2}\Big|_0^R - \frac{1}{r}\Big|_R^\infty\right] =$$

$$\frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{2R} + \frac{1}{R}\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0} \frac{1}{R} \frac{3}{2} \; ; \quad W_{\text{ionization}} = \frac{3}{8} \frac{e^2}{\pi\varepsilon_0} \frac{1}{R}$$

$$R = \frac{3}{8} \frac{e^2}{\pi\varepsilon_0} \frac{1}{W_{\text{ion}}}$$

$$R = \frac{3}{8\pi} \cdot \frac{\cancel{1{,}6 \times 10^{-19}}}{8{,}85 \cdot 10^{-12}} \quad \underbrace{\frac{\quad}{13{,}6\,eV \cdot \cancel{1{,}6 \cdot 10^{-19}}}}_{\text{to get Joules}}$$

$$= 1{,}587 \cdot 10^{-10}\ m = 1{,}59\ \text{Å}$$ radius of H from Thompson model.

II Estimate the number α particles will be scattered in angle range $44°$ to $46°$ if we fire 10^4 particles with the E of 1 MeV, into Cu (copper) foil with thickness $l = 5\ \mu m$.
Calculate the effective scattering cross section.

SOLUTION

$$\frac{dN}{N} = l \cdot n \cdot \sigma_\theta$$ (dN → number of particles scattered for a certain angle range.)

$$dN = N \cdot n \cdot l \left(\frac{Ze^2}{4\pi\varepsilon_0 M v^2} \right)^2 \frac{2\pi \sin\theta\, d\theta}{\sin^4 \frac{\theta}{2}}$$

in this problem, due to small angle range, we can take $45°$ as a constant and simply multiply w/ $d\theta = 2°$ and no integration needed. Normally you would tho.

$$\rho_{cu} = 8{,}9\ \frac{g}{cm^3}$$

$$A = 64\ \frac{g}{mol}$$

$$Z = 29$$

$$n = \frac{\rho N_A}{A}$$ how many nuclei per cubic meter

$$= \frac{8{,}9}{64} \cdot 6{,}02 \times 10^{23}$$

$$= 8{,}372 \cdot 10^{28}\ m^{-3}$$

$$\shortparallel$$

$$10^{22}\ cm^{-3}$$

$\theta \cong 45°$

$\sin \frac{45°}{2} \cong 0.38$

$d\theta = 2° = \frac{2}{57} = 0.035$

$\frac{mv^2}{2}$ KE of α particle. $= 1 MeV = 1.6 \times 10^{-13} J$

$dN =$ 13,2 particles.

$\Rightarrow$ cross section is a probability that is attenuation related (what?)

$$\frac{dN}{N} = \underset{\substack{\text{length of the foil, how long the scattering thing is}}}{\overset{[m]}{l}} \cdot \overset{m^{-3}}{n} \cdot \underbrace{d\sigma_\theta}_{\text{likelihood of scattering at } \theta} \rightarrow [\text{area}]$$

therefore,

$\sigma \Rightarrow$ describes the interaction of particles, or radiation

higher probability, higher the area of cross section

But cross section is not a probability since it has the measurements of area m^2.

Wtf is it then?

$$\frac{dN}{N} = \rho \cdot n \cdot d\sigma_{st}$$

$$d\sigma = \frac{13.2}{10^4 \cdot 5 \cdot 10^{-6} \cdot 8.37 \cdot 10^{28}} = 0.31 \cdot 10^{-26} m^2$$

conversion = $0.31 \cdot 10^{-22}$ cm^2

$m^2 \to cm^2$

$3.1 \cdot 10^{-23}$ cm^2

what is 1 barn?

$1 \text{ barn} = 10^{-24}$ cm^2

$d\sigma = 31$ barn.

III in H spectrum, two Ballmer series lines have wavelengths $\lambda_1 = 410.2\,nm$
$\lambda_2 = 486.1\,nm$

which series ($m = ?$) will contain the combination line?

SOLUTION

Ballmer series =

$$\bar{\nu} = R\left(\frac{1}{\underset{(two)}{2^2}} - \frac{1}{n^2}\right) \qquad \bar{\nu} = \frac{1}{\lambda}$$

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right); \quad \frac{1}{n^2} = \frac{1}{4} - \frac{1}{R\lambda} =$$

$$= \frac{R\lambda - 4}{4R\lambda} \Rightarrow n^2 = \frac{4R\lambda}{R\lambda - 4}$$

$$n_1 = \frac{4R\lambda_1}{R\lambda_1 - 4}, \quad n_2 = \frac{4R\lambda_2}{R\lambda_2 - 4}$$

$$= 6 \qquad\qquad = 4$$

combination line =

$$\bar{\nu} = R\left(\frac{1}{4^2} - \frac{1}{6^2}\right)$$

$n = 4$ // $\Rightarrow$ acknowledge, (Brochett)