ATOMIC AND NUCLEAR PHYSICS

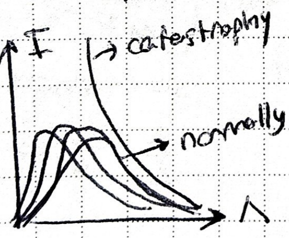
...
1895 x-rays => Wilhelm Rontgen
1896 radioactivity => Henri Becquerel
1897 electron
...

$1\,eV \Rightarrow \quad E = eU$

eV is for microworld
and joule is for macroworld.

- Charles Wilson
- Townsend
- Bragg
- Max Born
- Paul Langevin
- Balthasar von der Pol

$E = k_B T$
Black-body radiation - cathastropy



with T, peak shifts.

$E = nhf$ => understand the shift from $\int \rightarrow \Sigma$

GENERAL RELATIVITY

Inertial reference frame:

Coordinate system defined by a body is called a reference frame;

If two coordinates systems have constant velocity respect to eachother; then they called inertial reference frame.

All inertial frames must have identical physical laws ⇒ Gallielan principle.

Assumptions:

- distances in space are absoluse
- mass is same
- ~~tene~~ time intervals are absolute

Implication:

- all inertial frames are equally good.

MAXWELL'S EQUATIONS

wave eq: $$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

⇒ you can remember from units

$$\left[\frac{1}{m^2} - \frac{1}{m^2/t^2} \cdot \frac{1}{t^2}\right]$$

so that $\frac{1}{c^2}$ goes for $\frac{\partial^2 E}{\partial t^2}$.

is satisfied by:

$$E = E_0 e^{i(kx-\omega t)}$$

$$i(E) = E_0 \sin(kx-\omega t)$$

c is the velocity that this wave is propogating.

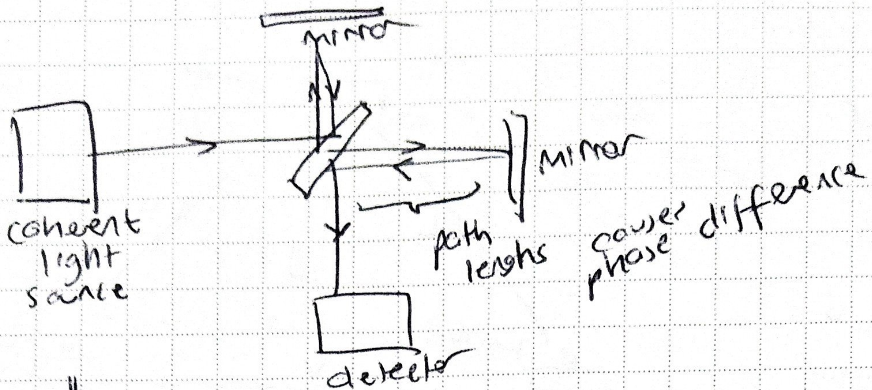
recall, $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

PROBLEM: which reference frame should be used to measure speed of light?

A reference frame where Maxwell's equations are the simplest? (?)

Michelson-Morley Experiment

what does it test out?



⇓

same frequency

Ether test = ⇒ Interference pattern should depend on the orientation of interferometer with respect to ether.

Didn't turn out that way...

Postulates of SR

1 physics laws are same for all inertial frames

2 speed of light is c. everywhere all the time.

$$c = \frac{\Delta x}{\Delta t}$$

Proof that c is constant:

Let's define Event.

Event (x, y, z, t).

Let's define simultaneity.

Wtf is it?

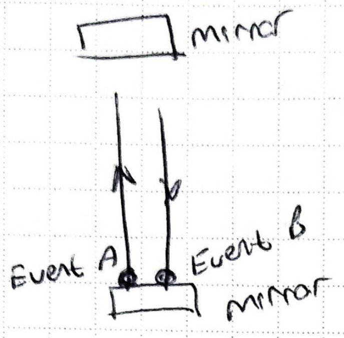
Let's define causality.

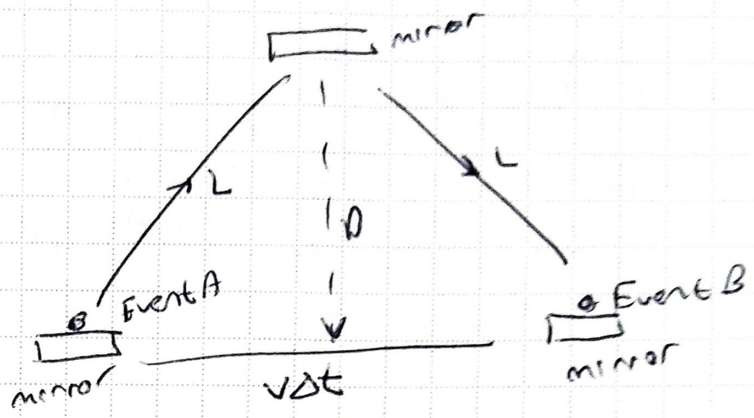
Cause and effect cannot swap in time

In STR, this is ensured by the fact that influence ...

TIME DILATION

lightclock





$$\Delta t = \frac{2L}{c} \qquad \text{here} \qquad L = \sqrt{\frac{1}{4}(v\Delta t)^2 + D^2}$$

$$D = \frac{\Delta t_0 \cdot c}{2}$$

$$L = \sqrt{\frac{1}{4}(v\Delta t)^2 + \frac{1}{4}(\Delta t_0 c)^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$ analyse...

here we define

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ Lorentz factor

$\beta = \frac{v}{c}$ if $\beta << 1$ classical mechanics will work.

time dilation

$$\Delta t_0 = \gamma \Delta t_0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Delta t_0$$

confirmed by =

MIC - muon lifetime μ^-, $\tau = 2,2 \mu s$
MAC - atomic clocks on planes

you can apply the time dilation formula to muon lifetime as if you accelarate it to $0,99c$ for example.

LENGTH CONTRACTION

Length contraction

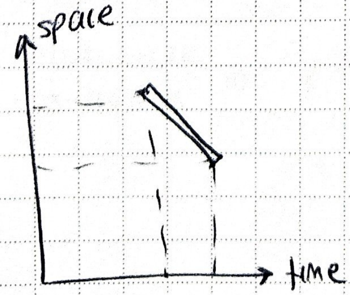
For an astronaut (Earth - Neptune)

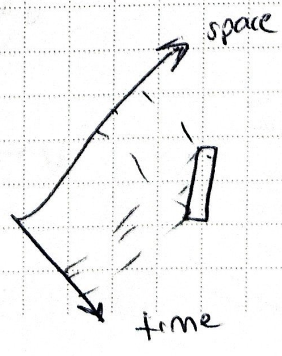
$$L = v \Delta t_0$$

from earth perspective, $L = v\Delta t_0 \sqrt{1-\frac{v^2}{c^2}}$

$$L = L_0 \sqrt{1-\frac{v^2}{c^2}}$$

four vector (x, y, z, ict) speed of light, time, imaginary





$$\Delta l = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \quad , \quad \Delta t$$

Galilean transformation $\Downarrow$ Lorentz transformation

In classical mechanics shifts between inertial reference frames the (Δl and Δt) were absolut-

$$I = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 t^2}$$

RELATIVISTIC MASS AND MOMENTUM

$p = mv$ or redefine it as $p = m\frac{\Delta x}{\Delta t}$

In the proper (unmoving) frame =

$$p = m_0 \frac{dx}{dt'} \quad , \quad dt' \Rightarrow \text{proper time}$$

$$p = m_0 \frac{dx}{dt'} = m_0 \frac{dt}{dt'} \frac{dx}{dt} = m_0 \frac{dt}{dt'} \underbrace{u}_{\text{velocity of the object}}$$

from time dilation =

$$dt = \gamma dt'$$

$$\frac{dt}{dt'} = \gamma$$

therefore,

$$\vec{p} = m_0 \gamma \vec{u} = m_0 \frac{\vec{u}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

compare $\vec{p} = m\vec{u}$

Relativistic mass =

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

you can analyse how mass will be

Now,

Newton's II Law

$$\vec{F} = m\vec{a} = = \frac{d\vec{p}}{dt} = \frac{d}{dt} m_0 \gamma \vec{u}$$

works in STR.

equally good in all inertial reference frames.

Now, what happens to the ENERGY?

if you recall, $KE = \frac{1}{2} mv^2$

RELATIVISTIC ENERGY

In newtonian physics, $KE = \frac{mv^2}{2} = \frac{p^2}{2m}$

Energy-work equivalance $(W = F.l)$

$$E_k = \int F.dx$$; Newton's 2nd Law

$$F = \frac{dp}{dt}$$

$\therefore$

$$E_k = \int \frac{dp}{dt} dx = \int dp \frac{dx}{dt} = \int v\, dp$$

relativistic momentum $\Rightarrow$ $p = m_0 \gamma v$

$$= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} v$$

$$E_k = \int v\,dp = \int v\,d(m_0\gamma v) = m\gamma^2 v^2 - \int m_0\gamma v\,dv$$

I.B.P.

$$= m_0\gamma v^2 - \frac{m_0}{2}\int \gamma\,d(v^2)$$

use the expansion of γ now.

$$E_k = m_0\gamma v^2 - \frac{m_0}{2}\int \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,d(v^2)$$

notice v^2 = a parameter

$$= m_0\gamma v^2 + \frac{m_0 c^2}{2}\int \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,d\left(1-\frac{v^2}{c^2}\right)$$

$$= m_0\gamma v^2 + m_0 c^2 \underbrace{\sqrt{1-\frac{v^2}{c^2}}}_{\frac{1}{\gamma}} - E_0$$

E_0 integration constant

$$E_k = m_0\gamma\left(v^2 + c^2\left(\sqrt{1-\frac{v^2}{c^2}}\right)^2\right) - E_0$$

$$E_k = m_0\gamma\left(v^2 + c^2\left(1-\frac{v^2}{c^2}\right)\right) - E_0$$

$$= m_0\gamma\,(\cancel{v^2} + c^2 \cancel{v^2}) - E_0$$

$E_k = m_0\gamma c^2 - E_0$ when

$u = 0$
$\gamma = 1$
$E_k = 0$

$E_0 = m_0 c^2$ REST ENERGY

$$E_k = m_0 \gamma c^2 - m_0 c^2$$

$$E_k = m_0 c^2 (\gamma - 1)$$

SUMMARY

$$E_0 = m_0 c^2$$

$$E_k = m_0 c^2 (\gamma - 1)$$

$$E = E_0 + E_k = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E^2 = p^2 c^2 + m_0 c^2 \quad **$$

TOTAL ENERGY

$$E = E_0 + E_k = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Now let's proof **

take E_{tot} and square it.

$$E^2 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}} = \frac{m_0^2 c^2 (c^2 + v^2 - v^2)}{1 - \frac{v^2}{c^2}}$$

$$= \underbrace{\frac{m_0^2 c^2 v^2}{1 - \frac{v^2}{c^2}}}_{} + \frac{m_0^2 c^4 \left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2}}$$

recall the relativistic momentum $\Rightarrow$ $p = \dfrac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

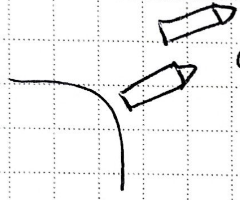
Relativistic energy - mass

Conclusion:

- anything is
- anything will non-zero mass will propogate with speed c.

Relativistic addition of velocities

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \quad **$$



0.6c respect to other rocket

0.6c respect to Earth

1.2c is not the answer...
Use = **

RELATIVISTIC ENERGY

$$E_k = m_0 c^2 (\gamma - 1)$$

If $v \ll c$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 + \frac{1}{2}\frac{v^2}{c^2} \ldots \Rightarrow$$ TAYLOR EXPANSION!

$$E_k = m_0 c^2 \frac{1}{2}\frac{v^2}{c^2} = \frac{m_0 v^2}{2}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \dots$$

TAYLOR EXPANSION

EXERCISES

① muon lifetime = $\tau = 2.2\ \mu s$

accelerated at: xc

"TIME DILATION"

$x = 0.9994$

$v = 0.9994$

SOLUTION

$$t = \gamma t'$$

$$t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = t = \frac{2.2\mu s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.2 \cdot 10^{-6} s}{\sqrt{1-\frac{0.9994^2 \cdot \cancel{c^2}}{\cancel{c^2}}}}$$

$$= \frac{2.2 \times 10^{-6} s}{\sqrt{1-0.9994^2}}$$

$$\frac{1}{\sqrt{1-0.9994^2}} = 28.87$$

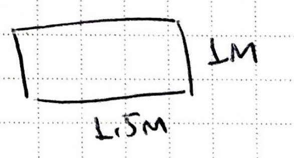
therefore the lifetime is,

$$\Delta t = t \cdot \gamma = 2.2 \times 28.87$$

$$= 63.5\ \mu s$$

② "LENGTH CONTRACTION"

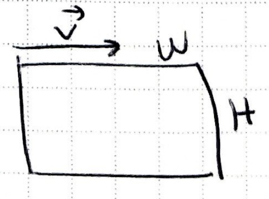
Picture the $(W \times H) = 1.5 \times 1\ M$



is in a spaceship flying at $v = 0.9c$

what are the dimensions for the Earth bound observer?

(Assume $\vec{v} \parallel W$)

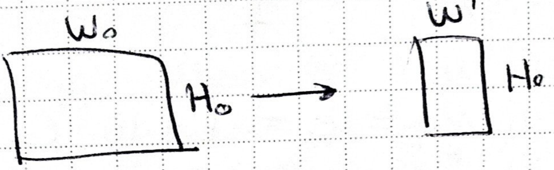


SOLUTION

$L_0 = 1.5m$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} \quad ; \quad L = 1.5\sqrt{1-0.81} = 0.654m$$

$1.5 \times 1 \rightarrow 0.65 \times 1$



③ "CONTINUE" At what speed does 500M - long train seem to be equal in length to 200M long tunnel?

$L_0 = 500M$; $L = 200M$ $v = ?$

SOLUTION

$\cancel{L = \gamma L_0}$

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{L^2}{L_0^2} = 1 - \frac{v^2}{c^2}$$

$$\frac{200^2}{500^2} = 1 - \frac{v^2}{c^2} =$$

$$\frac{v^2}{c^2} = 1 - \frac{L^2}{L_0^2}$$

$$v^2 = \left(1 - \frac{L^2}{L_0^2}\right)c^2$$

$$v = \left(\sqrt{1-\frac{L^2}{L_0^2}}\right)c \Rightarrow v \cong 0.92c$$

(4) In CRT (Cathode Ray Tube) television, electrons are accelerated by 25kV voltage. Calculate their relativistic velocity and compare to classical.

SOLUTION

Classical $\Rightarrow$ $$qU = \overbrace{\frac{m_e v^2}{2}}^{\text{classical KE}}$$

$$q = e = 1.6 \cdot 10^{-19} C$$

$$v_{classical} = \sqrt{\frac{2eU}{m_e}}$$

$$m_e = 9.1 \times 10^{-31} kg$$

$v_{classical} = 9.38 \cdot 10^{7}$ m/s

STR $\Rightarrow$ $$E_k = \underbrace{m_0 c^2 (\gamma - 1)}_{\text{STR KE}} = qU$$

$$\gamma = 1.049 = \frac{eU}{m_0 c^2} + 1$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.049$$

$$\rightarrow \left(\frac{1}{1.049}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$v_{rel} = \sqrt{1 - \left(\frac{1}{1.049}\right)^2}\, c = 9.06 \cdot 10^{7}$$

we can calculate the percentage difference,

$$\frac{v_{classical} - v_{rel}}{v_{rel}} \cong 3.5\%$$

Point checker: HOW TO TELL YOUR PROBLEM IS RELATIVISTIC OR CLASSICAL?

you can γ, if it doesn't deviate far from 1, you go classical.

You can also check energies.

(5) Find the velocity of a particle for its rest energy to be equal to its KE.

SOLUTION

$$m_0 c^2 (\gamma - 1) = m_0 c^2 \quad ?$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2 \quad \cancel{\boxed{v < c}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$v^2 = \frac{3}{4} c^2$$

$$v = \frac{\sqrt{3}}{2} c$$

6. A statistical Lithuanian uses about 2 MWh of energy per year. How much mass would satisfies a Lithuaina's annual requirement.

SOLUTION

$$1\,\text{MWh} = 1\,\text{MW} \cdot 3600 = 3600\,\text{MJ}$$

$; \; N = 3 \cdot 10^6$ (number of people)

$$\text{Total Energy} = E = N \cdot 3{,}6 \cdot 10^9\,\text{J} \overset{2\times}{=} 1{,}08 \cdot 10^{16}\,\text{J}$$

$$E = m_0 c^2$$

$$m_0 = \frac{E}{c^2} = 2\times\frac{1{,}08\times10^{16}\,\text{J}}{9 \cdot 10^{16}}$$

$$= \frac{2}{9} \cong 0{,}2\,\text{kg}$$

200 grams of matter would satisfy 1 Lithuanian's year long energy