03/03/2026 Nuclear Physics

recall the $dN \sin^4\frac{\theta}{2}$ vs. angle graph (from slides)
understand the graph

recall the Bohr's model postulates

II $L = n\hbar$

III $\hbar\omega_{mn} = E_n - E_m$

$$E_n = 13.6\,\text{eV}\, Z^2 \frac{1}{n^2}$$

Correspondence principle for H

Take 2 E levels such that
$E_1, E_2 \gg \Delta E = E_2 - E_1$

this condition leads to E levels to be no longer quantized.

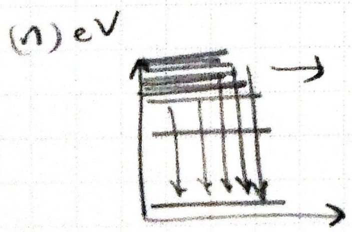
$$E_n = 2\pi\hbar c\frac{R}{n^2}, \quad \text{assume } n \gg 1$$

$$\Delta E_n = E(n+\Delta n) - E(n) = 2\pi\hbar Rc\frac{\Delta n(2n+\Delta n)}{n^2(n+\Delta n)^2}$$

$n \gg \Delta n$

is equivalent to say, $\Delta E_n \approx 2\pi\hbar c R\frac{2\Delta n}{n^3}$

you can view it in:



→ $n\uparrow$ energy gaps shrinks.
gets smaller.

Radiation frequency

classically $L = I\omega = n\hbar$ $\quad \omega = \frac{n\hbar}{I} = \frac{n\hbar}{mr_n^2}$, $r_n = \frac{4\pi\epsilon_0\hbar^2}{mZe}n^2$

$$\underset{\substack{\downarrow \\ ?}}{w_{re}} = \frac{mZ^2e^2}{(4\pi\varepsilon_0)^2\hbar^3n^3}$$ or using R,

w.freq
i think, no $w_{re} \Rightarrow$ figure out? $w_{classical}$.

using R: $$w_{re} = \frac{4\pi c R Z^2}{n^3}$$

Q Mechanically

$$\tilde{\nu} = RZ^2\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \qquad w_{QM} = 2\pi c\, \tilde{\nu}_{QM}$$

$$w_{QM} = 2\pi c R Z^2\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = 2\pi c R Z^2 \frac{(n-k)(n+k)}{n^2k^2}$$

neighboring energies $n = k+1$, $k = n-1$,

$n >> 1$

$$w_{QM} = 2\pi c R Z^2 \frac{2n-1}{n^2(n-1)^2}$$

$$w_{QM} = 2\pi c R Z^2, \frac{1}{n^3} \quad ?$$

Problems with Bohr's theory:

...

recall,

$\vec{L} = \vec{r} \times \vec{p}$

what does it mean to quatize angular momentum of H atom E levels?

It is ~~due~~ explained to external magnetic moments.
Recall the coils from EM theory. Similar principle.

quantizing angular momentum =

$$|\vec{L}| = \hbar\sqrt{\ell(\ell+1)} \qquad \ell = 0, 1, 2 \ldots n-1$$

$\ell = 0$ means electron orbiting its orbit $n=1$ without angular momentum!
What is that mean?

Here, also magnetic moment occurs due to electron rotation.

classical E dynamics =

$$\vec{p}_m = \underbrace{\frac{-|e|}{2m}}_{\text{magnetogyric ratio}} \vec{L}$$

another quantization =

$$L_z = m_\ell \hbar \qquad m_\ell = 0, \pm 1, \pm 2, \pm 3 \ldots \pm \ell$$

$$p_m = gL$$

$$L_z = m_\ell \hbar$$

Example =

$\ell = 1 \qquad |\vec{L}| = \hbar\sqrt{2}$

$\hbar\sqrt{\ell(\ell+1)}$

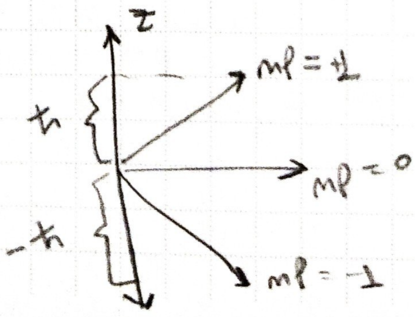
$m_\ell = -1, 0, 1$

then,

$$L_z = \hbar\{-1, 0, 1\}$$

Now how to plot =

Stern-Gerlach will be performed

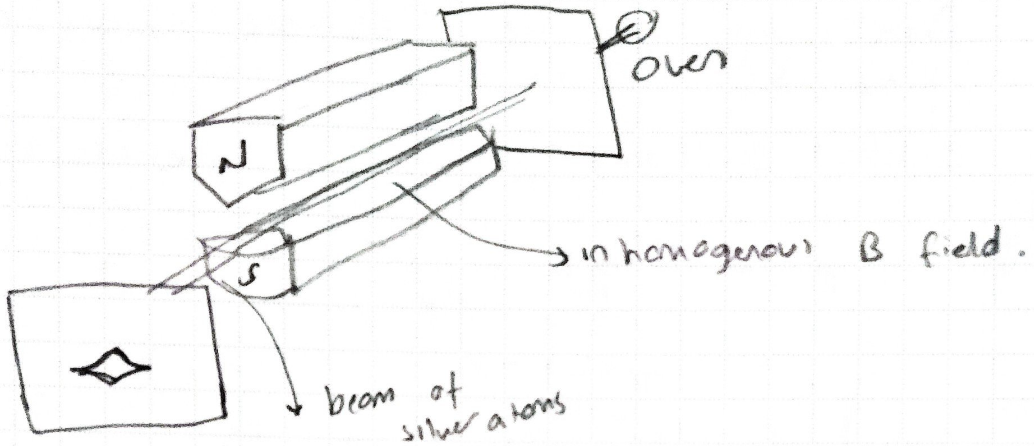


descreate.

Stern-Gerlach experiment: Spin

1922 ⇒ spin
1914 ⇒ bohr
1905 ⇒ relativity

we use non-uniform magnetic field because: to be able to move electrons (small magnets) and make them seperate, pull down or up. A uniform field would only rotate them but not pull!



classical expectation.

experimental result.

$$F \cong \mu_z \frac{\partial B}{\partial z}$$

Now, $|\vec{L}| = \hbar\sqrt{l(l+1)}$

Atoms with single e^-, with $l=0$ produce 2 lines.

Splitting magnitude $\mu_{mz} = \mu_B$

2 lines meant: $m_l = 1/2$!!! $l = \begin{matrix} 0 \\ 1 \end{matrix}$ → SPIN.

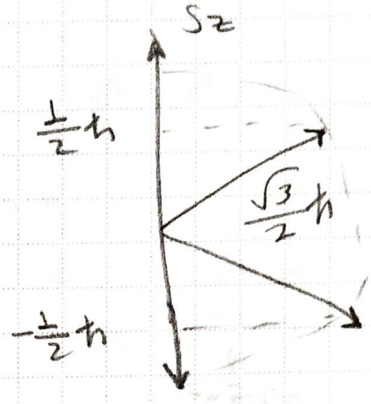
n spin = 1/2
e spin = 1/2
photon spin = 1 ≡ polarization!!!

Spin intrinsic angular momentum

$$|\vec{S}| = \hbar \sqrt{s(s+1)}$$

$s = 1/2$ for e^-

$$|\vec{S}| = \hbar \sqrt{\frac{3}{4}}$$



$$m_s = \frac{1}{2}, -\frac{1}{2}$$

$$S_z = \hbar m_s$$

the splitting in the screen, shows the how big the magnetic dipole is.

Magnetogyric ratio for spin is 2x larger than for orbital angular momentum.

$$g_s = \frac{e}{m}$$

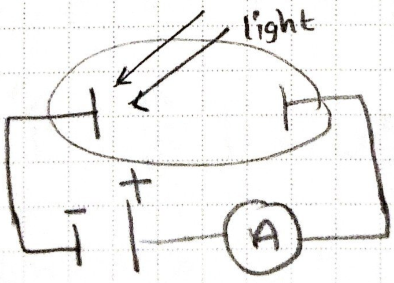
$$g_l = \frac{l}{2m}$$

Particle like properties of radiation

~~Photoelectric effect~~

1) Planck; $E = nhf$, $n = 0, 1, 2 ...$ 1900

2) Photoelectric effect,

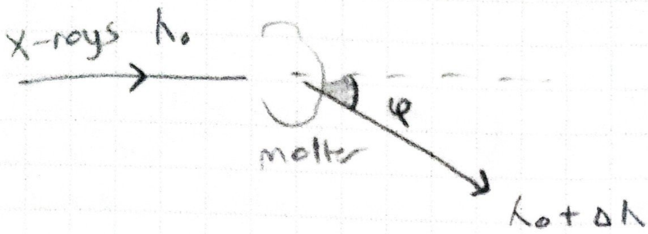


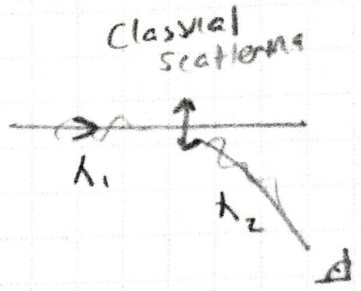
a dependence of wavelength in the measured current was detected.

$$hf = W + \frac{mv^2}{2}$$

W → work function

Compton Effect





$\lambda_1 = \lambda_2$

$\lambda_i \neq \lambda_f$

Photon moves @ c

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad m_0 = 0 \qquad \frac{(0)v}{\sqrt{1-1}} = \frac{0}{0}$$

compton suggested

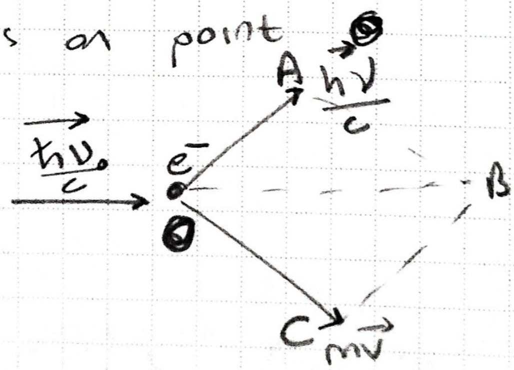
trick =

assume: $E = mc^2$, $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$

$$p = \frac{E v}{c^2} = \frac{h\nu}{c}$$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

e^- is on point



e^- momentum after collusion:

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$\beta^2 = \frac{v^2}{c^2}$$

Energy conservation:

$$h\nu_0 + m_0 c^2 = h\nu + \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$

momentum conservation:

$$\frac{\overrightarrow{\hbar \nu_0}}{c} = \frac{\overrightarrow{h\nu}}{c} + m\vec{v}$$

we'll use cosine theorem:

ΔOAB:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$m^2v^2 = \frac{h^2\nu_0^2}{c^2} + \frac{h^2\nu^2}{c^2} - \frac{2\hbar^2\nu_0\nu}{c^2}\cos\varphi$$

now we don't have to worry about the directions, magnitutes and angle is involved in this expression

$$m^2v^2c^2 = \hbar^2\nu_0^2 + \hbar^2\nu^2 - 2\hbar^2\nu_0\nu\cos\varphi \quad (1)$$

$$mc^2 = \hbar(\nu_0 - \nu) + m_0c^2 \quad (2)$$

apply $(2)^2 - (1)$ =

$$m^2c^4 = h^2\nu_0^2 - 2\hbar^2\nu_0\nu + \hbar^2\nu^2 + 2\hbar(\nu_0-\nu)m_0c^2 + m_0^2c^4$$

$$m^2c^4 - m^2v^2c^2 = -2\hbar^2\nu_0\nu + 2\hbar(\nu_0-\nu)m_0c^2 + m_0^2c^4 + 2\hbar^2\nu_0\nu\cos\varphi$$

$$m^2c^4\left(1-\frac{v^2}{c^2}\right) = 2\hbar^2\nu_0\nu\left(\cos\varphi - 1\right) + 2\hbar(\nu_0-\nu)m_0c^2 + m_0^2c^4$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_0 = m\sqrt{1-v^2/c^2}$$

$$\nu_0 - \nu = \frac{h\nu_0\nu}{m_0c^2}\left(1-\cos\varphi\right) = -\Delta\nu$$

where did β go?

Now using

$$\lambda = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda}$$

$$\frac{c}{\lambda_0} - \frac{c}{\lambda} = \frac{h}{m_0} \frac{1}{\lambda_0} \frac{1}{\lambda} (1 - \cos\varphi)$$

using $\Rightarrow$ $1 = \cos^2\alpha + \sin^2\alpha$

$$\cos\varphi = \cos^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}$$

$$\Delta\lambda = \lambda - \lambda_0 = 2\underbrace{\frac{\hbar}{m_0 c}}\sin^2\frac{\varphi}{2}$$ COMPTON FORMULA

$$\Lambda = \frac{\hbar}{m_0 c} \Rightarrow$$ compton wavelengt (2.42 pm)

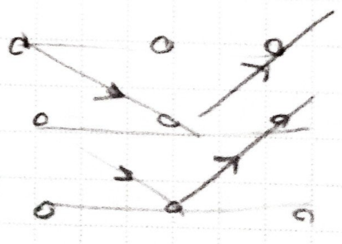
Answer = for a given voltages what is the smallest / longest wavelength you can get? of the x-ray?

$$E = eU$$

and $\frac{hc}{\lambda} = eU$

$\lambda = \frac{hc}{eU}$ is the condition.

How you measure x-ray wavelength?
Via diffraction grating!
How we form the grating?
what should be the period in that grating?
Crystals help!



$$\Delta\lambda = 2\Lambda \sin^2\left(\frac{\varphi}{2}\right)$$

de Broglie hypothesis 1924

If light is a particle, each particle can also be wave?

$$E = \hbar\nu - \hbar\omega$$

$$p = \frac{\hbar\nu}{c} = \frac{\hbar}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

recall:

$$\frac{2\pi}{\lambda} = k$$

$$\vec{p} = \hbar\vec{k}$$

$$\vec{E} = \vec{E_0} \exp(i(\omega t - (\vec{k}\cdot\vec{r})))$$

a plane wave expression

$E = \mathcal{E}$ field, $E \equiv$ energy (notation)

$$\vec{\mathcal{E}} = \mathcal{E}_0 \exp\left(\frac{i}{\hbar}(E \cdot t - \vec{p}\cdot\vec{r})\right) :$$

now here, all parameters can be associated with a particle

E = energy

p = momentum

r = coord.

$\underbrace{\qquad}$

particle...

light $\equiv$ particle

Exercises =

Calculate the velocity of e^- in a H atom (ground state), use bohr's model.

(in the exam def a question asking calculate this using
1 this model
2 this model)

II Law = $F = ma$

$$\frac{m v_n^2}{r_n} = \frac{Z e^2}{4\pi\varepsilon_0 r_n^2} \quad ; \quad r_n = 4\pi\varepsilon_0 \frac{\hbar^2 n^2}{m Z e^2}$$

$$v_n = \frac{Z e^2}{4\pi\varepsilon_0 m r_n} = \frac{Z^2 e^4}{(4\pi\varepsilon_0 \hbar n)^2} \quad ; \quad v_n = \frac{Z e^2}{4\pi\varepsilon_0 \hbar n},$$

ground state H =

$Z = 1$
$n = 1$

$\frac{e^2}{4\pi\varepsilon_0 \hbar}$ is a constant.

$$v_1 = \frac{(1.6 \times 10^{-19})^2}{4\pi \cdot 8.85 \cdot 10^{-12} \cdot 1{,}055 \cdot 10^{-34}} = 2.18 \cdot 10^{6} \text{ m/s}$$

$$\frac{v}{c} = \frac{2{,}18 \cdot 10^6}{3 \cdot 10^8} \cong \frac{1}{137} = \alpha$$

also, fine structure constant

a constant describing how relativistic an electron is!
uncalculable from first principles?

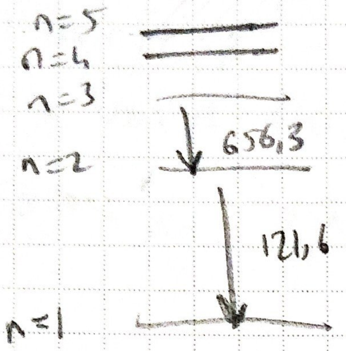
② Find the quantum number n of H atom orbit (state) given that relaxation of that state produced two photons with

$\lambda_1 = 656{,}3\ nm \rightarrow$ first line of balmer series

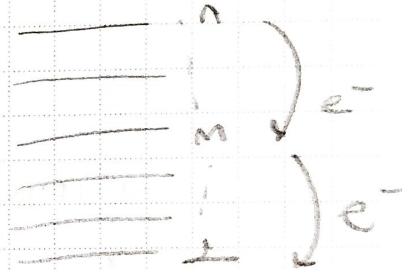
$\lambda_2 = 121.6\ nm$

SOLUTION

this question, you can ~~guess~~ estimate where relaxation occured, but normally it won't be.



$$\tilde{\nu}_1 = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$$



$$\tilde{\nu}_2 = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\tilde{\nu}_1 + \tilde{\nu}_2 = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\frac{\lambda_2 + \lambda_1}{\lambda_1 \lambda_2} = R\left(1 - \frac{1}{n^2}\right)$$

given: $\lambda_2 = 121.6\ nm$

$$\frac{1}{n^2} = 1 - \frac{\lambda_2 + \lambda_1}{R\lambda_1\lambda_2}$$

$$n = \sqrt{\frac{1}{1 - \frac{\lambda_2 + \lambda_1}{R\lambda_1\lambda_2}}} = n = 3$$

③ He atom.

He ionization energy is 24.6 eV. How much energy is required to completely ionize He → He^{++}

SOLUTION

$$E_n = 13.6eV \frac{1}{n^2} Z^2 \quad \text{for H}$$

$$\Downarrow$$

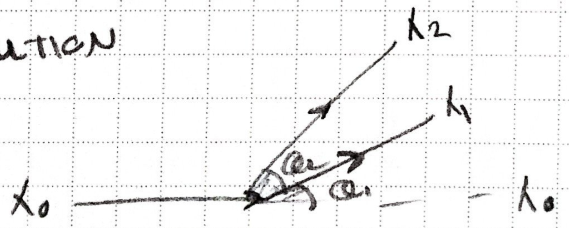
4H? WHY?

distinction is 24.6eV is for the 1st e^-, for the 2nd e^-, ionization E is different.

$$E_{total} = \underbrace{24.6}_{1st\ e^-} + \underbrace{54.4eV}_{2nd\ e^-} = 79.0$$

③ X-rays with wavelengths λ_0 are scattered by matter. Find λ_0 given that rays scattered @ $\theta_1 = 60°$, $\theta_2 = 120°$ have wavelengths different by the factor of 2.

SOLUTION



$$\Delta\lambda = 2\Lambda \sin^2\left(\frac{\varphi}{2}\right)$$

$\lambda_2 > \lambda_1 > \lambda_0$ right?

$$\frac{\lambda_0 + 2\Lambda \sin^2\left(\frac{60°}{2}\right)}{\lambda_0 + 2\Lambda \sin^2\left(\frac{120°}{2}\right)} = \frac{1}{2}$$

$$\lambda_0 = 2\Lambda \sin^2 60° - 4\Lambda \sin^2 30° = 2\Lambda\left(\frac{3}{4} - \frac{2}{4}\right) = \frac{\Lambda}{2}$$

since $\Lambda = 2{,}42\,pm$

$$\lambda_0 = \frac{\Lambda}{2} = 1{,}21\,pm = 1{,}21 \cdot 10^{-12}\,m. \;//$$

5. A photon with $\lambda_0 = 3{,}64\,pm$ is scattered by a free electron that obtains 25% of the incident photon energy. Find the change in photon wavelength and scattering angle.

SOLUTION

$$\eta = 0{,}25 = \frac{\hbar\Delta\nu}{h\nu_0} = \frac{\Delta\nu}{\nu_0} = \frac{\nu_0 - \nu}{\nu_0} = 1 - \frac{\nu}{\nu_0}$$

$$= 1 - \frac{\lambda_0}{\lambda} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda}$$

$$\eta\lambda_0 + \eta\Delta\lambda = \Delta\lambda$$

$$\Delta\lambda = \frac{\eta}{1-\eta}\lambda_0$$

$$\Delta\lambda = 2\Lambda \sin^2\frac{\theta}{2}$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{\Delta\lambda}{2\Lambda}}$$

$$\Delta\lambda = \frac{0{,}25}{0{,}75}\lambda_0 = 1{,}213\,pm$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{\Delta\lambda}{2\Delta}}$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{\Delta\lambda}{2\Delta}} = \sqrt{\frac{1{,}215}{2\cdot 2{,}4}} \approx 0{,}503$$

$$\frac{\theta}{2} = \operatorname{asin}(0{,}503) \approx 30° \Rightarrow \theta \simeq 60°$$